MJC 2, Integrals - PowerPoint

Q: $\displaystyle\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \frac{\pi}{12}$. Prove.

Solⁿ: $$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}} = \int_{\pi/6}^{\pi/2} \frac{\sqrt{\cos x}\,dx}{\sqrt{\cos x}+\sqrt{\sin x}} \quad -(1)$$

By Prop. $$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx.$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/3+\pi/6-x)}\;dx}{\sqrt{\cos(\frac{\pi}{2}-x)} + \sqrt{\sin(\frac{\pi}{2}-x)}}$$

$$\pi/3 + \pi/6 = \frac{9\pi}{18} = \pi/2$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}\;dx}{\sqrt{\sin x} + \sqrt{\cos x}} \quad \text{--- (2)}$$

(1) + (2)

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$$

$$2I = \int_{\pi/6}^{\pi/3} dx = x\Big]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{3\pi}{18} = \frac{\pi}{6}$$

or $I = \dfrac{\pi}{12}$ (Hence, proved).

2. $$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$