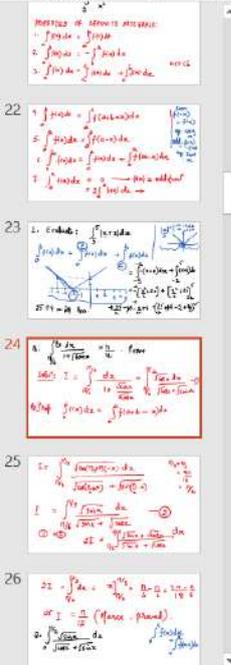


Q: $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \frac{\pi}{12}$. Prove.

Solun: $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \frac{\sqrt{\sin x}}{\sqrt{\cos x}}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x} dx}{\sqrt{\cos x} + \sqrt{\sin x}} \quad \text{--- (1)}$

By Prop. $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$.



$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/3 + \pi/6 - x)}}{\sqrt{\cos(\frac{\pi}{2} - x)} + \sqrt{\sin(\frac{\pi}{2} - x)}} dx$$

$$\pi/3 + \pi/6 = \frac{9\pi}{6} = \frac{3\pi}{2} = \pi/2$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \text{--- (2)}$$

$$\textcircled{1} + \textcircled{2} \quad 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

PROPERTIES OF INTEGRALS

- $\int f(x) dx + \int g(x) dx = \int (f(x) + g(x)) dx$
- $\int f(x) dx - \int g(x) dx = \int (f(x) - g(x)) dx$
- $\int k f(x) dx = k \int f(x) dx$

22. $\int f(x) dx + \int f(x) dx = 2 \int f(x) dx$

23. $\int f(x) dx + \int f(x) dx = 2 \int f(x) dx$

24. $\int f(x) dx = \int f(x) dx$

25. $\int f(x) dx = \int f(x) dx$

26. $\int f(x) dx = \int f(x) dx$

- 22. $\int_0^{\pi/3} \sin x dx = \int_0^{\pi/3} \cos x dx$
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$$2I = \int_{\pi/4}^{\pi/3} dx = x \Big|_{\pi/4}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

$\therefore I = \frac{\pi}{12}$ (Hence, proved).

$$2. \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$